Stress-Strain Relations of Reinforced Plastics Parallel and Normal to Their Internal Filaments

BERNARD W SHAFFER*
New York University, New York, N Y

The stress-strain relations and moduli of elasticity of filament reinforced plastics are derived in the direction of and normal to their internal filaments Consideration is given to the effect of plastic flow

Introduction

WITHIN recent years, considerable interest has developed in filament-reinforced plastics This new, composite material possesses many advantageous material properties including a high strength-to-weight ratio, which compares favorably with homogeneous metallic materials At one time, the fabrication of filament reinforced plastic pressure vessels presented itself as a formidable problem, but the difficulties have since been overcome There still remain some problems of analysis, but these too are being solved

Conventional elasticity formulas are not applicable for use in analyzing filament-reinforced plastics because some of the material property assumptions of classical elasticity are invalid Instead, it is necessary to derive new relations based on suitable simplifying assumptions In recognition of this fact, Kitzmiller, DeHaven, and Young[1] assumed in their analysis of a filament reinforced plastic pressure vessel that the resin is used as a binder with no force carrying capacity Their approach, commonly referred to as the netting analysis, was subsequently extended by Zickel[2] and Read[3] In a later analysis of the problem Hoffman[4] included the force carrying capacity of the resin and derived formulas which contain orthotropic material constants

The material constants of the composite material depend upon the properties and distribution of the filament and the resin Changes in its composition will affect the material constants in a way that may be studied experimentally, analytically, or by a combination of the two The analytic point of view will be developed in the present paper

Bell,[5] Brown,[6] Ekvall,[7] and Outwater,[8] to name a few, were motivated toward similar objectives All but one confined his analytical work to materials loaded in the direction of the reinforcing filaments They found relations similar to those presented at the beginning of the present paper Ekvall, on the other hand, also considered the complementary problem in which an external force was applied normal to the direction of the reinforcing filaments

It is the purpose of the present paper to analyze the behavior of filament reinforced plastics from a solid mechanics point of view The study is directed toward an evaluation of the stress strain relations in the direction of and normal to the internal filaments Each analysis will be conducted with the aid of an equivalent model Unlike the work of the previous investigators, the present study will not be confined to the elastic range, but will include the elastic-plastic range as well To the best of the author's knowledge, Hoffman[4] is the only other investigator who has heretofore used a plastic yield condition in his analysis of a reinforced plastic

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* Professor of Mechanical Engineering Associate Fellow Member AIAA

It will be assumed in the present analysis that the resin obeys the stress-strain relation shown in Fig 1 The stress-strain relation is linear in the elastic range where it obeys Hooke's law, and may deform at constant stress once its yield stress Y has been reached The numerical value of Y to be used in the analysis includes size effect, which reflects the very large ratio of resin surface to volume present in a reinforced plastic because of its many internal filaments Once the yield stress is reached, the true behavior of the resin is really open to speculation It may yield, fracture, or display a combination of both characteristics Yet, because we will be concerned only with strains of the order of magnitude of elastic strains, it is reasonable to assume that some straining beyond the yield stress can occur without requiring any additional load

The filaments are elastic and obey Hooke's law up to the yield stress Y_f The yield stress of some filament materials may be affected by the addition of a thin resin coating which seals microscopic surface imperfections and inhibits the initiation of surface cracks Should this be the situation, then the numerical value of Y_f should take this fact into consideration Otherwise Y_f represents the yield stress of the filament without any surface treatment The filament behavior beyond its yield stress is not specified because the analysis terminates once Y_f is reached

The analysis is divided into two parts First, we discuss the behavior of reinforced plastics in the direction of their internal filaments, and then we discuss its behavior normal to the internal filaments

Filaments Parallel to an Applied Force

In order to evaluate the behavior of reinforced plastics in the direction of its internal filaments, one can hypothetically combine all filaments and all resin into two separate areas and replace the actual material with the model shown in Fig 2 The model consists of two parallel bars joined to rigid end plates One bar of cross-sectional area A_f has a modulus of elasticity E_f and a yield stress Y_f equal to that of the filament The other bar of cross-sectional area A has a modulus of elasticity E and a yield stress Y equal to that of the resin The cross-sectional areas A_f and A correspond to the filament and resin cross-sectional areas, respectively They are chosen so that the percentage of filament by volume in the model and in the specimen are equal

The model is designed so that when a force P is applied to the end plates, it separates into P_f the force in the filament and P the force in the resin, as it does in the actual material The corresponding axial strain of the composite is equal to the axial strain of the filament bar and equal to the axial strain of the resin bar Thus it can be found that

$$\sigma_f = \left(\frac{E_f}{E}\right)\sigma = \frac{P/A_f}{1 + (E/E_f)(A/A_f)} \qquad (1)$$

where σ_f and σ are the stresses in the filament and in the resin,

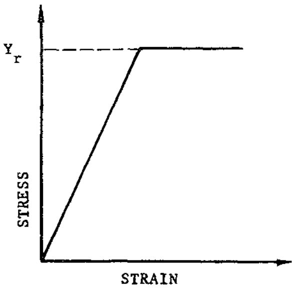

Fig 1 The stress-strain relation of resin

respectively Since the modulus of elasticity of the filament is greater than that of the resin, the stress in the filament is greater than that of the resin as long as both elements remain elastic

As the external force on the model increases, the stress in each bar also increases The stress in the resin reaches its yield stress first as long as the yield stresses and moduli of elasticity are related by the inequality

$$Y_f/Y > E_f/E \tag{2}$$

Inspection of the material properties of filament and resin currently used in filament reinforced plastics, shows that the previous inequality is always satisfied

At the instant the yield stress of the resin is reached σ is equal to Y, the total applied force is designated P_1, and the corresponding nominal stress

$$\sigma_1 = \frac{P_1}{A} = Y\left[1 + \frac{A_f}{A}\left(\frac{E_f}{E} - 1\right)\right] \tag{3}$$

where use has been made of the fact that the sum of A plus A_f is equal to the total cross-sectional area A The associated nominal strain ϵ_1, numerically equal to the strain of the resin, is equal to Y/E Thus, the modulus of elasticity of the filament reinforced plastic, equal to the ratio of nominal stress to nominal strain, may be written†

$$E = \frac{\sigma_1}{\epsilon_1} = E\left[\frac{A_f}{A}\frac{E_f}{E} + \left(1 - \frac{A_f}{A}\right)\right] \tag{4}$$

It is equal to the slope of the linear stress-strain relation shown on Fig 3 from the origin to the point marked 1 The previous expression shows that the modulus of elasticity of a filament reinforced plastic is directly proportional to the modulus of elasticity of the resin, the percentage of filament, and the moduli of elasticity ratio E_f/E The difference between the modulus of elasticity of the filament and that of the reinforced plastic is

$$E_f - E = [1 - (A_f/A)](E_f - E) \tag{5}$$

Since both terms in parentheses are positive, it is apparent that a reinforced plastic has a smaller modulus of elasticity than the filament it contains The difference between the moduli of elasticity $E_f - E$ is directly proportional to $(1 - A_f/A)$, the percentage of resin present in the filament-reinforced plastic

Once the yield stress of the resin is reached, the force in the resin remains constant at $Y A$, while the force in the filament P_f increases with an increase in the external force P The stress in the filament

$$\sigma_f = P_f/A_f = (P/A_f) - Y(A/A_f) \tag{6}$$

will continue to increase until Y_f is reached The corresponding nominal stress

$$\sigma_2 = P_2/A = Y + (A_f/A)(Y_f - Y) \tag{7}$$

and the nominal strain ϵ_2 is equal to Y_f/E_f

† Equation (4) is sometimes referred to as the "law of mixtures" Paul[9] showed that it provides an upper bound to E_s in those cases where both constituent materials have the same value of Poisson s ratio

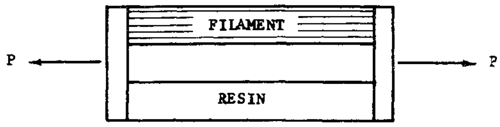

Fig 2 A model for studying reinforced plastics with filaments parallel to an applied force

The stress and strain at the instant the filament yields is depicted in Fig 3 by the coordinate of point 2 The nominal stress strain relation is linear between points 1 and 2 with a slope

$$C = (\sigma_2 - \sigma_1)/(\epsilon_2 - \epsilon_1) = (A_f/A)E_f \tag{8}$$

The previous relation was obtained by making use of Eqs (3) and (7) as well as the fact that ϵ_1 and ϵ_2 are equal to Y/E and Y_f/E_f, respectively Inspection of Eqs (4) and (8) shows that $E - C$ is positive so that E is always larger than C

The relative position of point 1 to point 2 on the stress-strain curve of Fig 3 may be of interest in evaluating the range over which the stresses are completely elastic It can be seen from Eqs (3) and (7) that as the ratios of yield stress and moduli of elasticity approach each other, point σ_1 approaches σ_2 and the section of the stress-strain curve from point 1 to point 2 decreases It completely disappears when the ratio Y_f/Y is equal to the ratio E_f/E, which, according to Eq (2), is the limit of applicability of the present solution

According to Eqs (4) and (8), the ratio $(E - C)/E$ indicates that the difference in slopes E and C may be minimized, if desired, by maximizing the percentage of filament and the ratio of moduli of elasticity E_f/E used in a reinforced plastic specimen It is also worth noticing that the difference in nominal strains $\epsilon_2 - \epsilon_1$, equal to $(Y_f/E_f) - (Y/E)$, is of the order of magnitude of elastic strains Thus the horizontal section of the stress strain relation of resin included in the present analysis is indeed small

Some interest also exists in the secant modulus of elasticity E', evaluated at the instant the filament reaches its yield stress The latter is equal to the slope of the line from the origin to point 2 on Fig 3 According to Eq (7) and the fact that ϵ_2 is equal to Y_f/E,

$$E' = \frac{\sigma_2}{\epsilon_2} = \frac{Y}{Y_f}\left[1 + \frac{A_f}{A}\left(\frac{Y_f}{Y} - 1\right)\right]E_f \tag{9}$$

It is interesting to observe that the secant modulus is a function of the yield stress of the filament and resin, whereas the modulus of elasticity E, defined by Eq (4), is a function of their moduli of elasticity Obviously, E' is less than E as long as the inequality of Eq (2) holds

Filaments Normal To An Applied Force

An enlarged cross section of reinforced plastic with a force P normal to the direction of the internal filament is shown in Fig 4 The filaments are of diameter d, equally spaced and separated by the resin

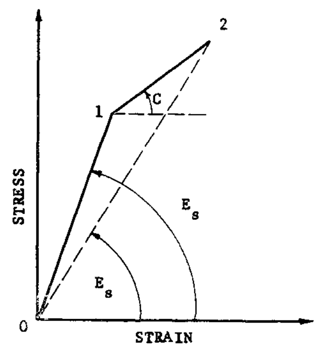

Fig 3 The stress-strain relation of filament reinforced plastic

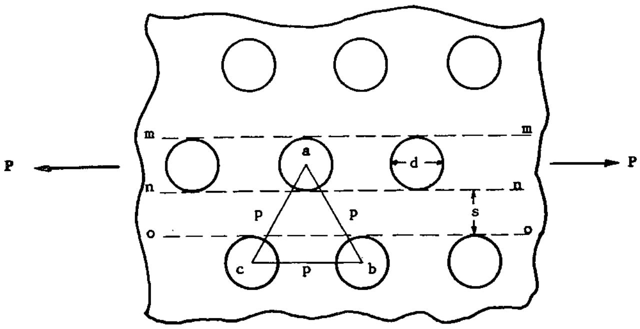

Fig 4 A cross section of the reinforced plastic with filaments normal to an applied force

Evaluation of the exact stress distribution in the specimen of Fig 4 would be extremely complicated, because from a mathematical point of view the resin is a multiply connected domain with statically indeterminate boundary conditions, while the filaments are subjected to statically indeterminate boundary conditions However, the situation is not hopeless, if initially one is content with a qualitative examination of the problem Then one can take an engineering approach and simplify the actual structure so that it becomes amenable to mathematical analysis The procedure is reasonable as long as the consequences are properly recognized and some attempt is made to balance the effect of major changes With the forementioned point of view in mind, let us replace the structure of Fig 4 with a relatively simple model, just as we did in the previous section

The model should account for a difference in the volume occupied by the filament and by the resin To find an expression for the area occupied by the filament, let us examine a unit thickness of the reinforced plastic shown in Fig 4 The sheet contains a uniform distribution of filaments of diameter d, spaced a distance p apart The triangular region marked abc is a typical region which is repeated throughout the specimen From the geometry of the region it can be seen that if A is the area of the triangle and A_f is the area occupied by the filament, the filament area ratio of the composite material

$$A_f/A = [\pi/2(3^{1/2})](d^2/p^2) \tag{10}$$

If one were to draw lines nn and oo tangent to the filaments, in the direction of the applied force, it would be observed that the lines are separated a distance

$$s = [(3)^{1/2}/2]p - d \tag{11}$$

The distance s is positive as long as A_f/A is less than 68% by volume The initial analysis will be conducted under the forementioned condition Then the analysis will be modified to describe the behavior of a reinforced plastic having more than 68% filament by volume

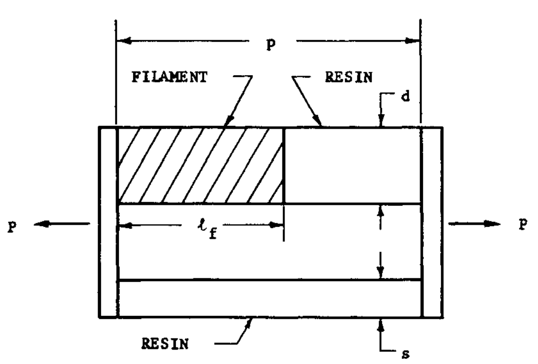

Fig 5 A model for studying reinforced plastics with filaments normal to an applied force

With the distance $s > 0$, one can imagine, in the direction of the external axial force, a flow of stress that lies entirely within the resin strips, and is uninterrupted by the filaments Most of the stress, however, does not flow without interruption from one external boundary to the other It continually meets and alternately crosses regions of resin and filament

To average the effect of the forementioned flow of stress, let us design our model with two bars, one of pure resin and the other consisting of filament and resin in series Such a model is shown in Fig 5 Each bar is of length p and of unit width perpendicular to the page The pure resin bar is marked r and is of width s The other bar, to be referred to as the equivalent bar, is marked e and is of width d The equivalent bar contains filament and resin in the same area ratio as its counterpart shown in Fig 4, within the region between the lines mm and nn Thus the length of filament in the model

$$l_f/p = (\pi d^2/4)/pd = \pi d/4p \tag{12}$$

The resin bar has a modulus of elasticity E and a yield stress Y The modulus of elasticity and yield stress of the equivalent bar is yet to be evaluated

The equivalent bar consists of two bars in series Thus, its modulus of elasticity is given by the expression

$$\frac{E_r}{E} = 1 - \frac{l_f}{p}\left(1 - \frac{E}{E_f}\right) \tag{13}$$

In view of Eq (12), it may also be written

$$\frac{E}{E} = 1 - \frac{\pi}{4}\frac{d}{p}\left(1 - \frac{E_r}{E_f}\right) \tag{14}$$

The modulus of elasticity of the equivalent bar is larger than that of the resin because $E_f > E$

A force applied to the model of Fig 5 distributes itself between P the force in the resin bar and P the force in the equivalent bar Each bar is equally strained, thus the stress in the equivalent bar

$$\sigma = (E/E)\sigma = P/d[1 + (s/d)(E/E)] \tag{15}$$

The stress in the equivalent bar is larger than that of the resin bar because $E > E$ It will, therefore, be the first to yield Furthermore, yielding will begin in the resin region of the equivalent bar, because the yield stress of the resin is less than that of the filament At yield, σ of Eq (15) is equal to Y and P is designated P_1 Then, in view of Eqs (11, 14, and 15), the nominal stress in the model σ_1 is given by the expression

$$\sigma_1 = \frac{P_1}{d + s} = Y\left[1 - \frac{\pi}{4}\frac{d}{p}\left(1 - \frac{E_r}{E_f}\right)\left(1 - \frac{2}{3^{1/2}}\frac{d}{p}\right)\right] \tag{16}$$

The associated nominal strain at the initiation of yield ϵ_1 is equal to the strain of the equivalent bar In view of Eq (12), and the fact that P is equal to $Y d$,

$$\epsilon_1 = \frac{Y_r}{E}\left[1 - \frac{\pi}{4}\frac{d}{p}\left(1 - \frac{E_r}{E_f}\right)\right] \tag{17}$$

The nominal stress-strain relation of the model, equal to that of the reinforced plastic loaded normal to the internal filaments, is shown in Fig 3 The slope of the line between the origin and point 1 is the modulus of elasticity of the reinforced plastic In view of Eqs (10, 16, and 17),

$$E = \frac{\sigma_1}{\epsilon_1} = \frac{E\left\{1 - \left(1 - \frac{E_r}{E_f}\right)\left[0\ 8247\left(\frac{A_f}{A}\right)^{1/2} - \frac{A_f}{A}\right]\right\}}{\left[1 - 0\ 8247\left(\frac{A_f}{A}\right)^{1/2}\left(1 - \frac{E_r}{E_f}\right)\right]} \tag{18}$$

The equation shows that, for a fixed value of E, the modulus of elasticity of a reinforced plastic normal to their internal filaments increases as either the percentage of filament increases or the ratio E/E_f decreases When $2d/3^{1/2}p$ is equal to unity, corresponding to a filament ratio of 68%, Eq (16) simplifies to $\sigma_1 = Y$, and the expression for E becomes simply

$$\frac{E_r}{E} = 1 - \frac{\pi 3^{1/2}}{8}\left(1 - \frac{E_r}{E_f}\right) \tag{19}$$

Once the external force applied to the model shown in Fig 5 is of sufficient intensity to cause yielding of the resin in the equivalent bar, the force within the equivalent bar remains constant at $Y\,d$ The stress in the resin bar is then related to the external force P in accordance with the relation

$$\sigma = (P/s) - (d/s)Y \tag{20}$$

As the external force P is increased, so is the stress in the resin bar The stress will increase until σ is equal to Y and P is equal to P_2 Then, according to Eqs (11) and (20), the external force on the model is

$$P_2 = (3^{1/2}/2)Y\,p \tag{21}$$

and the nominal stress

$$\sigma_2 = P_2/(s + d) = Y \tag{22}$$

The nominal strain ϵ_2 then corresponds to the strain of the resin bar and is equal to Y/E

The associated stress strain relation is shown on Fig 3 as the straight line from point 1 to point 2 In view of Eqs (16, 17, and 22), as well as the fact that ϵ_2 is equal to Y/E, the slope C of this line may be expressed as

$$C = \frac{\sigma_2 - \sigma_1}{\epsilon_2 - \epsilon_1} = \left(1 - \frac{2}{3^{1/2}}\frac{d}{p}\right)E_r \tag{23}$$

The previous expression is applicable as long as there exists a positive distance s, as shown in Fig 4 When s vanishes, the term in the parentheses also vanishes, as one can see by checking with Eq (11) and Eq (23)

The force that can be carried beyond initial yield is given by Eqs (11, 16, and 21) as

$$P_2 - P_1 = \frac{\pi}{4}\left(1 - \frac{E_r}{E_f}\right)\left(\frac{3^{1/2}}{2} - \frac{d}{p}\right) Y\,d \tag{24}$$

It is relatively small but may be increased by decreasing the modulus ratio E/E_f and increasing the yield stress of the resin It disappears entirely when the filament ratio is 68% for then $(3^{1/2}/2 - d/p)$ vanishes

The secant modulus associated with point 2 on Fig 3 is equal to the ratio of Eq (22) to the nominal strain ϵ_2, with ϵ_2 equal to Y/E Thus, the secant modulus at point 2 is equal to the modulus of elasticity of the resin and is independent of all the other material properties

The previous analysis applies when there exists less than 68% filament by volume When more filament exists, the model of Fig 5 should be modified by removing the bar of pure resin The appropriate model is then only a single bar with filament and resin in series Its modulus of elasticity was previously expressed by Eq (13) When more than 68% filament exists, the length ratio l_f/p is equal to the area ratio A_f/A, and Eq (13) should be rewritten to read

$$\frac{E}{E} = 1 - \frac{A_f}{A}\left(1 - \frac{E_r}{E_f}\right) \tag{25}$$

Under an externally applied force, the resin and filament are equally stressed Again the resin is the first to yield This time, however, there is no secondary member to restrain the elongation, so that the model can extend without any additional external force until fracture occurs Thus, there is no discontinuity of slope in the stress-strain curve when a reinforced plastic contains more than 68% of filament by volume

Discussion of Results

An analysis has been presented which describes the stress-strain relations of reinforced plastics parallel to or normal to their internal filaments To simplify what would otherwise be an extremely complex analysis, the actual materials were replaced by equivalent models whose physical characteristics are similar to those of the reinforced plastics Even though the models were carefully selected, some limitations exist that restrict the use of the results that were obtained These limitations arise by virtue of the fact that substitution of the models for the actual material leads to an evaluation of an average stress instead of the actual stress distribution Nevertheless, the external results that were obtained, such as the shape of the stress-strain curves and the expressions for the moduli of elasticity, do represent a good engineering approximation of the actual situation On the other hand, details of the internal results, such as a description of the stress field or the stresses at which the stress-strain curves change slope, are not nearly as accurate and should be used only from a qualitative viewpoint

There have been some reports published in the open literature which point to the reasonableness of the results obtained For example, Kitzmiller, DeHaven, and Young[1] have presented experimental evidence which shows the stress-strain curve of spiralloy‡ to be of the general form shown in Fig 3 Yet other investigators have expressed the belief, privately, that the stress-strain curve is linear up to failure The discrepancy may now be resolved by virtue of the fact that the present analysis shows that either possibility may occur, depending upon the material composition Furthermore, since the present analysis shows the upper region of the curve to be relatively small for some reinforced plastics, it is suggested that additional data be taken to identify the shape of the curve in the yield region before a positive conclusion is reached

It is felt that sufficient data are available to confirm the reasonableness of the present theory with regard to its prediction of the moduli of elasticity of a filament reinforced plastic For example, if we chose as representative values E equal to $0\ 4 \times 10^6$ and E_f equal to 10×10^6, Eq (4) shows that the resulting modulus of elasticity of a reinforced plastic with 68% filament by volume should be $6\ 93 \times 10^6$ in the direction of internal filaments, whereas Eq (19) shows that it should be only $1\ 15 \times 10^6$ normal to the direction of the internal filaments If, on the other hand, we assume that the modulus of elasticity of the resin is $0\ 35 \times 10^6$ while the ratio between that of the filament and resin remains 25, the computed moduli of elasticity become $6\ 06 \times 10^6$ and $1\ 01 \times 10^6$, respectively

Equations for the internal stress distribution are not to be used for quantitative purposes Nevertheless, we should not overlook its revelation concerning the distribution of stress in a simple tensile specimen § To illustrate the point, compare the stresses in two test specimens with different filament orientation but with the same material properties, E equal to $0\ 4 \times 10^6$ and E_f equal to 10×10^6 According to Eq (1), the filament stress in a specimen with filaments parallel to the applied load is 25 times that of the resin during the initial stages of loading On the other hand, the filament stress is only 2 88 times the stress in the interrupted section of the resin when the filaments are normal to the applied load In each case the stress distribution is significantly nonuniform even though the applied strain is uniform From the relative magnitude of the computed stress differences, it is apparent that the deviation from a uniform

‡ Spiralloy is Hercules Powder Company's registered trade mark for filament reinforced plastics

§ The nonuniformity of the stress field was also discussed by J C Schulz in a comment to Ref 4

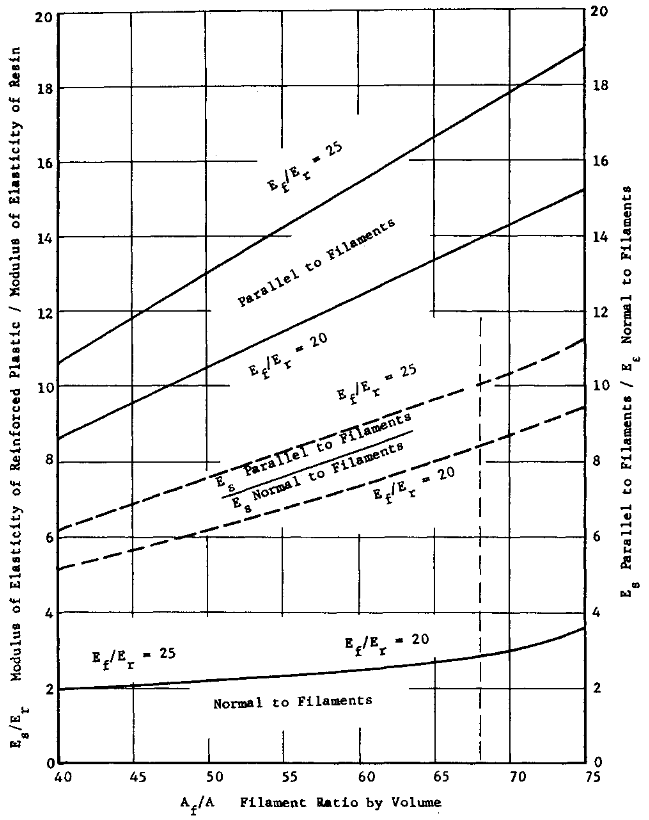

Fig 6 The relation between the moduli of elasticity of reinforced plastics and their filament ratio by volume

stress field is also highly dependent upon the filament orientation

Let us now study some of the relations between the moduli of elasticity of a reinforced plastic and its material properties, from a graphical representation of Eqs (4, 18, and 25) The relations are shown in Fig 6, where the filament ratio by volume is taken as the abscissa whereas the moduli of elasticity ratio of the reinforced plastic to its resin is taken as the ordinate It can be seen that, for a fixed resin, the moduli of elasticity of reinforced plastics increase with the percentage of filament to resin A relatively small difference in the ratio E_f/E causes a significant separation of the curves that represent the behavior of the composite in the direction of their internal filaments No significant separation appears when the external force is normal to the internal filaments

It may be interesting to compare the moduli of elasticity in the direction of and normal to the internal filaments of otherwise identical reinforced plastics This may be done by computing the ratios of Eq (4) to Eq (18) and Eq (4) to Eq (25) The results are also shown in Fig 6 It is found that with less than 68% filament by volume the ratios are almost linear between A_f/A equal to 0 40 and 0 68 To a very reasonable degree of accuracy, one can approximate the ratio

$$E_{p\ \ ll\ l}/E_{n\ \ m\ l} = 11\ 15(A_f/A) + 0\ 58 \qquad (26a)$$

for E_f/E = 20 and

$$E_{p\ \ allel}/E_{n\ \ m\ l} = 13\ 61(A_f/A) + 0\ 73 \qquad (26b)$$

for E_f/E = 25

As indicated earlier in this section, the description of the internal stress distribution should be viewed with caution The description may serve as a first approximation for reinforced plastics in the direction of their internal filaments, but no such statement can be made for the other case At most, the stress analysis of reinforced plastics normal to their internal filaments may be used for qualitative discussions

The stress analysis of the previous section shows two different situations that may occur, depending upon the filament ratios When the filament ratio is relatively small, the model of Fig 5 indicates yielding of the resin in the sections between filaments, before the clear strip of resin of width s reaches its yield stress The stress-strain curve was found to consist of two linear sections which meet at a point that corresponds to the attainment of the resin yield stress in the equivalent bar No such sharp deviation is expected in actuality Instead, as individual particles of resin between filaments continuously reach their yield stress, the slope of the stress-strain curve changes continuously until all the resin eventually yields

Throughout the present analysis, it was tacitly assumed that the resin can be strained beyond the yield stress by an amount which is of the order of magnitude of elastic stresses In the event that resins are used which do not obey the latter requirement, the analysis of events between points 0 and 1 of Fig 3 will not change, but those between points 1 and 2 will be affected The change will be small and will result in a decrease of the slope C For reinforced plastics, with filaments that lie parallel to the applied force, the slope C will become equal to E_f, the modulus of elasticity of the filament; but the slope C will become equal to E for reinforced plastics with filaments that lie normal to the applied force

It was also tacitly assumed that the bond stress between the filament and resin is greater than the yield stress of the resin No change in the analysis of reinforced plastics in the direction of their internal filaments would occur if this were not true However, when an external force is applied normal to the internal filaments of a reinforced plastic, the slope of the stress-strain curve changes from E to E as separation takes place between the resin and the filament because the equivalent bar of Fig 5 can no longer support any tensile force

The models of Fig 2 and Fig 5 were used to study the stress-strain relations of reinforced plastics under a uniaxial tensile force They may also be used to study its behavior under a uniaxial compressive force

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